PG – 619

II Semester M.Sc. Degree Examination, July 2017
(RNS – Repeaters) (2011-12 and Onwards)
MATHEMATICS
M-202 : Complex Analysis

Time : 3 Hours   Max. Marks : 80

Instructions : *i) Answer* ***any five full*** *questions choosing atleast* ***two*** *from* ***each*** *Part.*
ii) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Define Mobius transformation. Prove that every Mobius transformation maps circles and straight lines in the z-plane in to circles or lines.

   b) State and prove the Cauchy's integral formula for derivative and use it to evaluate $\int_{|z|=3} \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$.

   c) If f(z) is continuous in an open set G in the complex plane and $\int_C f(z)\, dz = 0$ for every simple closed curve in G, then prove that, the function f(z) is analytic on G. **(6+6+4)**

2. a) State and prove Cauchy's theorem for a rectangle.

   b) State and prove Liouville's theorem. Deduce the fundamental theorem of Algebra. **(8+8)**

3. a) Find the radius of convergence of

   i) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n} (z-2i)^n$

   ii) $\sum_{n=0}^{\infty} (\log n)^n z^n$.

P.T.O.

b) Define radius of convergence of a power series. If R is the radius of convergence of $\sum a_n z^n$ then prove the following.

i) The power series converges for $|z| < R$ and diverges for $|z| \geq R$.

ii) If $0 < \rho < R$ the power series converges uniformly in $\{|z| \leq \rho < R\}$.

c) State and prove Taylor's theorem for an analytic function f(z) in a region D about the point z = 0 in D. **(4+6+6)**

4. a) Find the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in

i) | z | < 2

ii) 2 < | z | < 3

iii) | z | > 3.

b) Let z = a be an essential singularity of an analytic function f(z) and K = { | z – a | < r } be an neighbourhood of 'a'. For a given $\in > 0$ and any complex number ξ, prove that there exists a point z with 0 < | z – a | < r such that $|f(z) - \xi| < \in$.

c) Let f(z) be analytic function having an isolated singularity at z = a. If | f(z) | is bounded in neighbourhood { 0 < | z – a | < r }. Then prove that f(z) has a removable singularity at z = a. **(5+6+5)**

PART – B

5. Evaluate the following :

a) $\int_0^{2\pi} \frac{d\theta}{a + b\sin\theta}$

b) $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$

c) $\int_{-\infty}^{\infty} \frac{\cos(ax)}{(x^2+b^2)^2}\,dx,\ a>0$

d) $\int_{-\infty}^{\infty} e^{-x^2}\cos(2mx)\,dx,\ m>0$. **(4+4+4+4)**

6. a) State and prove Argument principle.

b) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2-3x+2)}$.

c) State and prove open mapping theorem. **(6+4+6)**

7. a) State and prove Phragmen Lindelof theorem.

b) State and prove Riemann mapping theorem.

c) Using the result of the Weierstrass factorization theorem, construct an entire function having zero's at 1, 2, 3. **(7+6+3)**

8. a) Let f(z) be analytic in the region $|z| < \rho$ and let $z = re^{i\theta}$ be any point of this region. Then prove that $f(re^{i\theta}) = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{(R^2-r^2)f(Re^{i\phi})\,d\phi}{R^2-2Rr\cos(\theta+\phi)+r^2}$.

b) Derive the Jensen's formula in standard notation. **(8+8)**